NTPL
ALL INDIA TEST SERIES

CODE - A
TEST ID 001916

JEE (Main) - 2019

FULL TEST - 1

Time : 3 Hours ***Maximum Marks : 360***

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **90 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Maths).
10. The test is of **3 hours** duration. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

Name of the Candidate (in Capitals) ____________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

PHYSICS

SECTION – I

This section contains *30 questions.* Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which *ONLY ONE* is correct. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

1. In an experiment, to measure the volume of an irregular lamina using screw gauge, a student observes that the zero line of the head scale is 4 division below the line of graduation. There are 100 divisions on the circular scale. He gave 2 complete rotations of the circular scale and found that the distance moved by the screw is 2 mm. If the main scale reading is 4, then find the relative percentage error in the volume. Given 35th division coincides with the main scale line and the measured length of the body has a relative error of 2.25%

(a) 1.63% (b) 2.71% (c) 3% (d) 1.93%

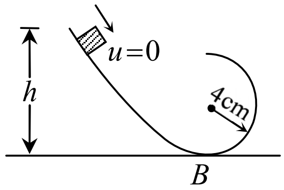
2. Two balls are thrown simultaneously from a top of a building with same initial velocity u. One ball is thrown vertically upwards and the other ball is thrown vertically downwards. The distance between the two bodies after time 5 s is (take, $g = 10\ m/s^2$)

(a) $10u$ (b) $5u$ (c) $15u$ (d) $20u$

3. Two balls A and B are projected simultaneously from the origin with initial velocities 50 m/s and 20 m/s at angle 60° and 30° respectively, with horizontal. The position of the ball A with respect to ball B after 3 s in y-direction is given by

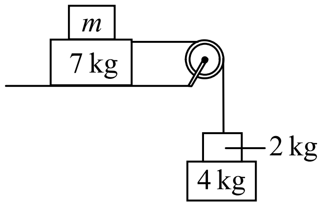
(a) 117.79 m (b) 112.30 m (c) 94.50 m (d) 100 m

4. A block of mass 4 kg is released from a height of 25 cm on a smooth track. The minimum value of h, so that it completes the vertical circle should be



(a) 25 cm
(b) 10 cm
(c) 20 cm
(d) 5 cm

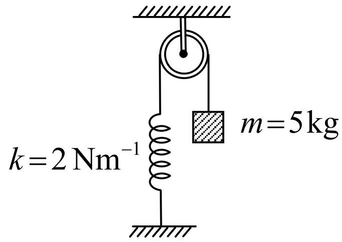
5. Four blocks are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown in figure. The coefficient of static friction of A with table is 0.50. The minimum mass m of the block, so that blocks do not move should be



(a) 5 kg
(b) 6 kg
(c) 4 kg
(d) 3 kg

6. A block of mass 5 kg is connected to a massless spring through a string passing over a frictionless pulley. Initially, the block is at rest and spring is in the state of its natural length. The maximum elongation produced in the string will be

(take, $g = 10\ \text{m/s}^2$)



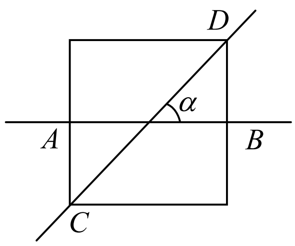
(a) 50 m

(b) 25 m

(c) 100 m

(d) 5 m

7. Consider a uniform square plate. Line AB passes through its centre. CD is another line passing through centre of plate, it makes an angle α with AB. If moment of inertia about an axis AB is $2I$, then the moment of inertia of the plate about the axis CD is



(a) $2I$

(b) $2I\sin^2\alpha$

(c) $2I\cos^2\alpha$

(d) $2I\cos^2(\alpha/2)$

8. A satellite of mass $2m$ is circulating around the earth with constant angular velocity. If the radius of the orbit is R_0 and mass of the earth is M, then the angular momentum about the centre of earth is

(a) $2m\sqrt{GMR_0}$ (b) $M\sqrt{2GmR_0}$ (c) $2m\sqrt{\dfrac{GM}{R_0}}$ (d) $M\sqrt{\dfrac{GM}{R_0}}$

9. If the pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake, then the depth of the lake is

(a) 10 m (b) 20 m (c) 60 m (d) 30 m

10. 3 kg of ice at –13°C is mixed with 10 kg of water at 13°C in an insulating vessel having a negligible heat capacity. The specific heat of water and ice are 1 kcal/kg/°C and 0.5 kcal/kg/°C, respectively. The latent heat of fusion of ice is 80 kcal/kg. The final mass of water remaining in the container is

(a) 11.4 kg (b) 10.7 kg (c) 7.2 kg (d) 9.3 kg

11. At 27°C, the ratio of density of a fixed mass of a gas to a pressure of gas is $\frac{1}{2}$. At 153°C, the ratio will be

(a) $\frac{1}{2}$ (b) $\frac{75}{213}$ (c) $\frac{213}{75}$ (d) $\frac{153}{300}$

12. A Carnot engine has efficiency $\frac{1}{6}$. Efficiency becomes $\frac{1}{3}$ when the temperature of the sink is lowered by 57K. What is the temperature of the source?

(a) 285 K (b) 171 K (c) 399 K (d) 342 K

13. A particle of mass m is executing oscillations about the origin on the X-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is A, then its time period T is

(a) proportional to $\frac{1}{\sqrt{A}}$ (b) independent of A (c) proportional to $\sqrt{A}$ (d) proportional to $A^{3/2}$

14. A closed hollow insulated cylinder is filled with gas at 27°C and also contains an insulated piston of negligible weight and negligible thickness at the middle point. The gas on one side of the piston is heated to 90°C. If the piston moves 10 cm, then the length of the hollow cylinder is

(a) 210.4 cm (b) 96.5 cm (c) 105.2 cm (d) 64.2 cm

15. A string is rigidly tied at two ends and its equation of vibration is given by $y = \cos 2\pi t \sin 2\pi x$. Then, minimum length of string is

(a) 1 m (b) $\frac{1}{2}$m (c) 5m (d) $2\pi m$

16. In a diode AM-detector, the output circuit consists of R = 1 kΩ and C =10 pF. A carrier signal of 100 kHz is to be detected. Is it good

(a) yes (b) no

(c) information is not sufficient (d) None of the above

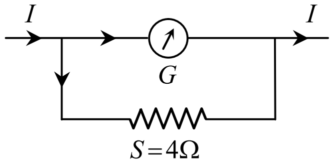
17. A parallel plate capacitor of plate area A and plate separation d is charged to potential V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitors so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system in the process of inserting the slab, then state correct relation from the following

(a) $Q = \frac{\varepsilon_0 AVK}{d}$ (b) $W = \frac{\varepsilon_0 AV^2}{2Kd}$ (c) $E = \frac{V}{Kd}$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left(1 + \frac{1}{K}\right)$

18. Two identical thin rings each of radius 2 m are coaxially placed at a distance 2 m apart. If 5 C and 10 C are respectively, the charges uniformly spread on the two rings, the work done in moving a charge of 3C from the centre of one ring to that of other is $\left(K = \frac{1}{4\pi\varepsilon_0}\right)$

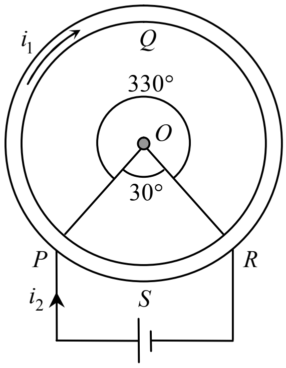
(a) 0 (b) + 2.20 k (c) 31.8 k (d) 38.41 k

19. When a shunt of 4Ω is attached to galvanometer, the deflection reduces to $\frac{1}{5}$th. If an additional shunt of 2Ω is attached, the deflection will be



(a) $I/5$
(b) $I/13$
(c) $I/10$
(d) $I/12$

20. A cell is connected between two points P and R of a circular conductor $PQRS$ of centre O with angle POR = 30°. If B_1 and B_2 are the magnitudes of the magnetic field at O due to the currents in PQR and PSR respectively, the ratio $\frac{B_1}{B_2}$ is



(a) 0.2
(b) 0.8
(c) 1.5
(d) 1

21. A 20 eV electron is circulating in a plane perpendicular to a uniform field of magnetic induction 3×10^{-5} Wb/m^2. The orbital radius of the electron is given by $\left(\sqrt{7.03} = 2.65\right)$

(a) 70 cm (b) 35 cm (c) 25 cm (d) 50 cm

22. A small circular loop of radius 2.5 cm is placed inside a square loop of edge 30 cm. The loops are coplanar and concentric. The mutual inductance will be, if current flowing through square loop is 5A.

(a) 6×10^{-11} H (b) 7.4×10^{-9} H (c) 7.4×10^{-5} H (d) 6.5×10^{-5} H

23. A star initially has 10^{40} deutrons. It produces energy via the processes.

$$^2_1\text{H} + ^2_1\text{H} \longrightarrow ^3_1\text{H} + \text{p}$$

$$^2_1\text{H} + ^3_1\text{H} \longrightarrow ^4_2\text{He} + \text{n}$$

The masses of the nuclei are as follows

M(H^2) = 2.014amu; M(p) = 1.007 amu

M(n) = 1.008 amu; M(He^4) = 4001 amu

If the average power radiated by the star is 10^{16}W, the deutron supply of the star is exhausted in a time of the order of

(a) 10^6 s (b) 10^8 s (c) 10^{12} s (d) 10^{16} s

24. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. The value of n will be

(a) 1 (b) 2 (c) S (d) 4

25. The half-life of Bi^{210} is 5 days. What time is taken by (7/ 8)th part of the sample to decay?

(a) 3.4 days (b) 10 days (c) 15 days (d) 20 days

26. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a

(a) 50 W blue lamp (b) 1 W infrared lamp (c) 50 W blue lamp (d) 1 W ultraviolet lamp

27. The transfer ratio of a transistor is 50. The transistor is used in common-emitter configuration. The input resistance is $4\,k\Omega$ and input AC voltage is 0.02 V. The peak value of collector current is

(a) 0.25 mA (b) 0.25 μA (c) 0.50 mA (d) 0.50 μA

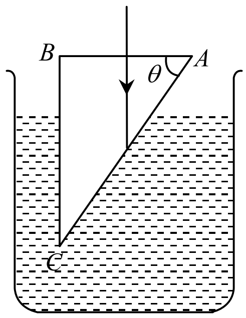
28. A lens is placed between a source of light and a wall. It forms images of area A_1 and A_2 on the wall for its two different positions. The area of the source or light is

(a) $\frac{A_1 + A_2}{2}$ (b) $\left[\frac{1}{A_1} + \frac{1}{A_2}\right]^{-1}$ (c) $\sqrt{A_1 A_2}$ (d) $\left[\frac{\sqrt{A_1} + \sqrt{A_2}}{2}\right]^2$

29. In a Young's double slit experiment, the separation between the slits is 0.5 mm. The distance between the screen and slit is 1.5 m. For a monochromatic light of wavelength 500nm. the distance of 3rd minima from the central maxima is

(a) 2.25 mm (b) 0.75 mm (c) 3.75 mm (d) 1.50 mm

30. A glass prism $(\mu = 1.5)$ is dipped in water $(\mu = 4/3)$ as shown in figure. A light ray is incident normally on the surface AB. It reaches the surface BC after totally reflected, if



(a) $\sin\theta \geq 8/9$

(b) $2/3 < \sin\theta < 8/9$

(c) $\sin\theta \leq 2/3$

(d) It is not possible

CHEMISTRY

SECTION – II

This section contains *30 questions.* Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which *ONLY ONE* is correct. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

31. Which of the following set of quantum numbers is correct for the 19th electron of chromium?

	n	l	m	s
(a)	3	0	0	1/2
(b)	3	1	–2	1/2
(c)	4	0	0	1/2
(d)	4	1	–1	1/2

32. The energy of an electron in the 3rd orbit of an atom is $-E$. The energy of an electron in the first orbit will be

(a) $-3E$ (b) $-\frac{E}{3}$ (c) $-\frac{E}{9}$ (d) $-9E$

33. Out of $N_2I, SO_2, I_3^+, I_3^-, H_2O, NO_2^-$ *and* N_3^- the linear species are

(a) NO_2^-, I_3^+, H_2O (b) N_2O, I_3^+, N_3^- (c) N_2O, I_3^-, N_3^- (d) N^{3-}, I^{3+}, SO_2

34. Planar structure is shown by

(a) CO_3^{2-} (b) BCl_3 (c) $N(SiH_3)_3$ (d) All of those

35. Peroxide ion ___

(i) Has five completely filled antibonding molecular orbitals

(ii) Is diamagnetic

(iii) Has bond order one

(iv) Is isoelectronic with neon

Which one of these is correct?

(a) (iv) and (iii) (b) (i), (ii) and (iv) (c) (ii), (iii) (d) None of these

36. Among KO_2, KO_2, AlO_2^-, BaO_2 and NO_2^+, unpaired electron is present in

(a) NO_2^+, BaO_2 (b) KO_2 and AlO_2^- (c) KO_2 only (d) BaO_2 only

37. All form ideal solutions except

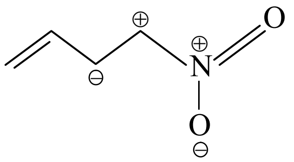
(a) C_2H_5Br and C_2H_5I (b) C_6H_5Cl and C_6H_5Br

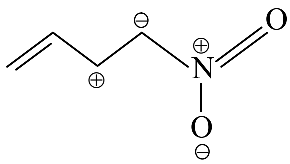
(c) C_6H_6 and $C_6H_5CH_3$ (d) C_2H_5I and C_2H_5OH

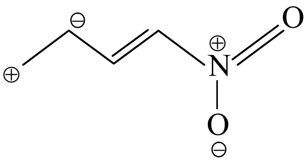
38. The units of constant a in vander Waals' equation is

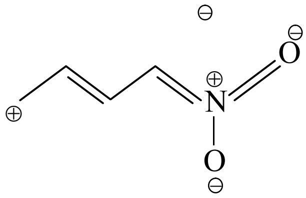
(a) $dm^6\ atm\ mol^{-2}$ (b) $dm^3\ atm\ mol^{-1}$ (c) $dm\ atm\ mol^{-1}$ (d) $atm\ mol^{-1}$

39. Among the following, the least stable resonance structure is

(a) 

(b) 

(c) 

(d) 

40. Match the List –I and List – II and pick the correct matching from the codes given below:

List-I (Atomic/Molecular Species)	List-II (Corresponding pairs)
A. Isotope	1. $^{228}Ra_{88}$ & $^{228}Ac_{89}$
B. Isobar	2. $^{39}Ar_{18}$ & $^{40}K_{19}$
C. Isotone	3. $^{2}H_1$ & $^{3}H_1$
D. Isosters	4. $^{235}U_{92}$ & $^{231}Th_{90}$
E. Isodiaphers	5. CO_2 & N_2O

	A	B	C	D	E
(a)	2	1	4	5	3
(b)	2	5	1	4	3
(c)	3	1	2	5	4
(d)	5	4	1	2	3

41. Amongst the following solutions, the buffer solution is

(a) $NH_4Cl + NH_4OH$ solution (b) $NH_4Cl + NaOH$ solution

(c) $NH_4OH + HCl$ solution (d) $NaOH + HCl$ solution

42. Which of the following oxides will not give OH^- in aqueous solution?

(a) Fe_2O_3 (b) MgO (c) Li_2O (d) K_2O

43. Saturated solution of KNO_3 is used to make 'salt-bridge' because

(a) Velocity of K^+ is greater than that of NO_3^-

(b) Velocity of NO_3^- is greater than that of K^+

(c) Velocities of both K^+ and NO_3^- are nearly the same

(d) KNO_3 is highly soluble in water

44. $\lambda_{ClCH_2COONa} = 224\, ohm^{-1}\ gmeq^{-1}$

$\lambda_{NaCl} = 38.2\, ohm^{-1}\, cm^2\ gmeq^{-1}$

$\lambda_{HCl} = 203\ ohm^{-1}\ cm^2\ gmeq^{-1}$

What is the value of λ_{ClCH_2COOH} ?

(a) $288.5\, ohm^{-1}\ cm^2\ gmeq^{-1}$
(b) $289.5\, ohm^{-1}\ cm^2\ gmeq^{-1}$
(c) $288.8\, ohm^{-1}\ cm^2\ gmeq^{-1}$
(d) $59.5\ ohm^{-1}\ cm^2\ gmeq^{-1}$

45. Gold numbers of protective colloids A, B C and D are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers are

(a) $C < B < D < A$
(b) $A < C < B < D$
(c) $B < D < A < C$
(d) $D < A < C < B$

46. Which of the following statement is correct with respect to the property of elements with an increase in atomic number in the carbon family (group 14)

(a) Atomic size decrease
(b) Ionisation energy increase
(c) Metallic character decrease
(d) Stability of +2 oxidation state increase

47. Match List I (molecules) with List II(Boiling points) and select the correct answer.

List I	List II
(A) NH_3	(i) 290 K
(B) PH_3	(ii) 211 K
(C) AsH_3	(iii) 186 K
(D) SbH_3	(iv) 264 K
(E) BiH_3	(v) 240 K

(a) $A-iii, B-ii, C-v, D-iv,\ E-i$
(b) $A-v, B-iii, C-ii, D-iv,\ E-i$
(b) $A-i, B-iv, C-v, D-ii,\ E-iii$
(d) None of these

48. KF combines with HF to form KHF_2 . The compound contains the species

(a) $K^+, F^-\ and\ H^+$
(b) $K^+, F^-\ and\ HF$
(c) $K^+\ and\ [HF_2]^-$
(d) $[KHF]^+\ and\ F^-$

49. A metal M reacts with N_2 to give a compound 'A' (M_3N). 'A' on heating at high temperature gives back 'M' and 'A' on reacting with H_2O gives a gas 'B'. 'B' turns $CuSO_4$ solution blue on passing through it. A and B can be

(a) Al and NH_3
(b) $Li\ and\ NH_3$
(c) $Na\ and\ NH_3$
(d) $Mg\ and\ NH_3$

50. In the test for nitrate ion, the brown ring formed has a formula $[Fe(H_2O)_5NO]SO_4$. The oxidation number of iron in this complex is

(a) + 1 (b) + 2 (c) + 3 (d) 0.

51. Which of the following complex will show geometrical as well as optical isomers?

(a) $[Pt(NH_3)_2Cl_2]$ (b) $[Pt(NH_3)Cl_5]$ (c) $[Pt(en)_3]^{4+}$ (d) $[Pt(en)_2Cl_2]$.

52. The following compounds have been arranged in order of their increasing thermal stabilities. Identify the correct order:

I. K_2CO_3 II. $MgCO_3$ III. $CaCO_3$ IV. $BeCO_3$

(a) I < II < III < IV (b) IV < II < III < I (c) IV < II < I < III (d) II < IV < III < I

53. Which of the following statements is correct regarding the slag obtained during the extraction of a metal like copper or iron?

(a) The slag is lighter and has higher melting point than the metal.

(b) The slag is lighter and has lower melting point than the metal.

(c) The slag is heavier and has higher melting point than the metal.

(d) The slag is heavier and has lower melting point than the metal.

54. Given below are a set of resonating structures and their stability order is provided in bracket. Select which one of the following is incorrectly matched.

(a) $\underset{(I)}{CH_2=CH-CH=CH_2} \longleftrightarrow \underset{(II)}{CH_2-\overset{+}{C}H=CH-\bar{C}H_2}$ $(I > II)$

(b) $\underset{(I)}{\overset{+}{C}H_2-\ddot{\underset{..}{O}}-CH_2} \longleftrightarrow \underset{(II)}{CH_2=\overset{+}{\underset{..}{O}}-CH_3}$ $(II > I)$

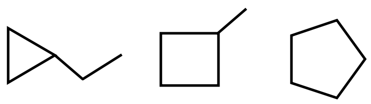
(c) $\underset{(I)}{CH_2=CH-\ddot{\underset{..}{Cl}}:} \longleftrightarrow \underset{(II)}{\bar{C}H_2-CH=\overset{+}{\ddot{C}}l:}$ $(II > I)$

(d) All of the above are correctly matched.

55. How many acyclic isomers can be drawn for the molecular formula C_3H_6O?

(a) 2 (b) 6 (c) 5 (d) 4

56. Rank the following compounds in order of increasing heat of combustion



(I) (II) (III)

(a) (II) < (I) < (III)

(b) (II) < (III) < (I)

(c) (I) < (II) < (III)

(d) (III) < (II) < (I)

57. Alkyl halides can be obtained by all methods except

(a) $CH_3CH_2OH + HCl/ZnCl_2 \longrightarrow$

(b) $CH_3–CH_2–CH_3–CH_2 \xrightarrow[Cl_2]{\text{UV light}}$

(c) $C_2H_5OH + NaCl \longrightarrow$

(d) $CH_3COOAg + Br_2/CCl_4 \longrightarrow$

58. Which of the following will not undergo aldol condensation?

(a) Acetaldehyde

(b) Propionaldehyde

(c) Trideuterio acetaldehyde

(d) Benzaldehyde

59. Which one of the following is NOT correct for sucrose:

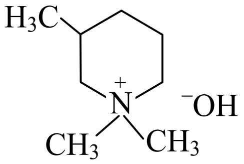
(a) Hydrolysis of sucrose with dilute acid yields an equimolar mixture of D-glucose and D-fructose.

(b) Acid hydrolysis of sucrose is accompanied by a change in optical rotation.

(c) The glycosidic linkage in sucrose is in between C-1 of glucose and C-2 of fructose.

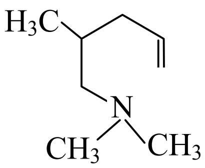
(d) Aqueous solution of sucrose exhibits mutarotation.

60. Hofmann's elimination product of A is:

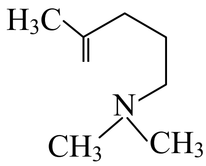


(A)

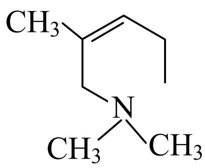
(a)



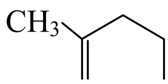
(b)



(c)



(d)



MATHS

SECTION – III

This section contains *30 questions.* Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which *ONLY ONE* is correct. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

61. The function of f and g are given by $f(x)=\{x\}$, the fractional part of x and $g(x)=\frac{1}{2}\sin[x]\pi$, where $[x]$ denote the integral part of x, then range of gof is

(a) $[-1, 1]$ (b) $[0]$ (c) $\{-1, 1\}$ (d) $[0, 1]$

62. The function $f(x)=\frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at $x=0$. The value which should be assigned to f at $x=0$ so that it is continuous there, is

(a) $a-b$ (b) $a+b$ (c) $\log a+\log b$ (d) none of these

63. The coefficients a and b that make the function $f(x)=\begin{cases} 1/|x| & \text{for} |x|\geq 1 \\ ax^2+b & |x|<1 \end{cases}$ differentiable at any point are given by

(a) $a=-1/2, b=3/2$ (b) $a=1/2, b=-3/2$ (c) $a=1, b=-1$ (d) $a=1/2, b=3/4$

64. If $y=\sqrt{\frac{1-\sin^{-1}x}{1+\sin^{-1}x}}$ then $y'(0)$ is equal to

(a) 1 (b) $1/2$ (c) -1 (d) $\sqrt{2}/3$

65. The function $f(x)=(\log(x-1))^2(x-1)^2$ has

(a) local extremum at $x=1$
(b) point of inflection at $x=1$
(c) local extremum at $x=2$
(d) point of inflection at $x=2$

66. If $\int_0^{x^2} f(t)\,dt = x\cos\pi x$, then the value of $f(4)$ is

(a) 1 (b) $\frac{1}{4}$ (c) -1 (d) $-\frac{1}{4}$

67. The solution of $\frac{x\,dy}{x^2+y^2}=\left(\frac{y}{x^2+y^2}-1\right)dx$ is

(a) $y=x\cot(c-x)$ (b) $\cos^{-1}y/x=-x+c$ (c) $y=x\tan(c-x)$ (d) $y^2/x^2=x\tan(c-x)$

68. If $I=\int_0^{\infty}\frac{\sqrt{x}dx}{(1+x)(2+x)(3+x)}$, then I equals

(a) $\frac{\pi}{2}\left(2\sqrt{2}-\sqrt{3}-1\right)$ (b) $\frac{\pi}{2}\left(2\sqrt{2}+\sqrt{3}-1\right)$ (c) $\frac{\pi}{2}\left(2\sqrt{2}-\sqrt{3}+1\right)$ (d) none of these

69. The length of the perpendicular from the points $(m^2,2m),(mm',m+m')$ and $(m'^2,2m')$ to the line $x+y+1=0$ from

(a) an A.P. (b) a G.P. (c) a H.P. (d) none of these

70. Three concentric circles of which the biggest is $x^2+y^2=1$ have their radii in A.P. with common difference $d(>0)$. If the line $y=x+1$ cuts all the circles in real distinct points, then

(a) $d>\frac{2-\sqrt{2}}{4}$ (b) $d>\frac{2+\sqrt{2}}{4}$ (c) $d>1+\frac{1}{\sqrt{2}}$ (d) d is any real number

71. Equation of a circle touching the line $|x-2|+|y-3|=4$ will be

(a) $(x-2)^2+(y-3)^2=12$ (b) $(x-2)^2+(y-3)^2=4$

(c) $(x-2)^2+(y-3)^2=10$ (d) $(x-2)^2+(y-3)^2=8$

72. Let $P=\{\theta:\sin\theta-\cos\theta=\sqrt{2}\cos\theta\}$ and $Q=\{\theta:\sin\theta+\cos\theta=\sqrt{2}\sin\theta\}$ be two sets. Then

(a) $P\subset Q$ and $Q-P\neq\varnothing$ (b) $Q\not\subset P$

(c) $P\not\subset Q$ (d) $P=Q$

73. Let P $(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ where $\theta+\phi=\pi/2$, be two points on the hyperbola $x^2/a^2-y^2/b^2=1$. If (h,k) is the point of intersection of normal at P and Q, then k is equal to

(a) $\frac{a^2+b^2}{a}$ (b) $-\left[\frac{a^2+b^2}{a}\right]$ (c) $\frac{a^2+b^2}{b}$ (d) $-\left[\frac{a^2+b^2}{b}\right]$

74. If the tangent at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $x^2/a^2+y^2/b^2=1$ meets the auxillary circle in two points, the chord joining them subtends a right angle at the centre; then the eccentricity of the ellipse is given by

(a) $(1+\cos^2\theta)^{-1/2}$ (b) $1+\sin^2\theta$ (c) $(1+\sin^2\theta)^{-1/2}$ (d) $1+\cos^2\theta$

75. The tangent at three points A, B and C on the parabola $y^2 = 4x$, taken in pairs intersect at the points P, Q and R. If Δ, Δ' be the areas of the triangles ABC and PQR respectively, then

(a) $\Delta = 2\Delta'$ (b) $\Delta' = 2\Delta$ (c) $\Delta = \Delta'$ (d) none of these

76. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is :

(a) 4 (b) 2 (c) 3 (d) 9

77. If z_1 and z_2 lie on the same side the line $\bar{a}z + \bar{a}z + b = 0$, where $a \in \mathbf{C}, a \neq 0, b \in \mathbf{R}$, then the ratio $\frac{az_1 + a\bar{z}_1 + b}{\bar{a}z_2 + a\bar{z}_2 + b}$ is

(a) purely imaginary (b) a positive real number

(c) a negative real number (d) none of these

78. Sum of the common roots of $z^{2006} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 1 = 0$ is

(a) 0 (b) -1 (c) 1 (d) 2

79. If $|a \pm b| > c$ and $a \neq 0$, then the quadratic equation

$a^2 x^2 + (b^2 + a^2 - c^2)x + b^2 = 0$

(a) has two real roots (b) both positive roots

(c) cannot have real roots (d) none of these

80. Let $S = \frac{4}{19} + \frac{44}{19^2} + \frac{444}{19^3} + ...$ upto ∞. Then S is equal to

(a) $40/9$ (b) $38/81$ (c) 36/171 (d) none of these

81. The value of λ for which the system of equations

$2x - y - 2z = 2,$

$x - 2y + z = -4$

$x + y + \lambda z = 4$

has no solution is

(a) 3 (b) -3 (c) 2 (d) -2

82. A group of 6 boys and 6 girls is randomly divided into two equal groups. The probability that each group contains 3 boys and 3 girls is

(a) 10/231 (b) 5/231 (c) 90/231 (d) 100/231

83. In a hurdle race, a runner has probability p of jumping over a specific hurdle. Given that in 5 trials, the runner succeeded 3 times, the conditional probability that the runner had succeeded in the first trial is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

84. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is :

(a) 243 (b) 729 (c) 64 (d) 2187

85. In triangle ABC,

$\frac{a\cos A+b\cos B+c\cos C}{a+b+c}$ is equal to

(a) $r+R$ (b) R/r (c) r/R (d) r/R

86. If $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}+\frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)}=0,$

then $\tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4$ is equal to

(a) –1 (b) 1 (c) 2 (d) 4

87. If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & -2 & 4\end{bmatrix}, 6A^{-1}=A^2+cA+dI, then\,(c,d)\,is$

(a) (–11, 6) (b) (–6, 11) (c) (6, 11) (d) (11, 6)

88. The value of a for which the volume of parallelopiped formed by the vectors $i+aj+k$, $j+ak$ and $ai+k$ is minimum is

(a) –3 (b) 3 (c) $1/\sqrt{3}$ (d) $-\sqrt{3}$

89. If θ is the angle between the line $r=2i+3j-k+(i+j+k)t$ and the plane $r.(3i-4j+5k)=q$, then

(a) $\cos\theta=\frac{2\sqrt{6}}{15}$ (b) $\sin\theta=\frac{2\sqrt{6}}{15}$ (c) $\cos\theta=-\frac{11\sqrt{7}}{70}$ (d) $\sin\theta=-\frac{11\sqrt{7}}{70}$

90. The lines $r=i-j+\lambda(2i+k)$ and $r=(2i-j)+\mu(i+j-k)$ intersect for

(a) $\lambda=1,\mu=1$ (b) $\lambda=2,\mu=3$

(c) all values of λ and μ (d) no value of λ and μ.

SOLUTION OF AITS JEE (MAIN) FULL TEST – 1

PHYSICS

1. **(b)**

$$\text{Pitch} = \frac{\text{Distance moved by the screw}}{\text{Number of complete rotations of the ciruclar scale}} = \frac{2\text{ mm}}{2} = 1\text{ mm}$$

Total number of divisions on the circular scale, $N = 100$

$\therefore$ Least count, $\frac{P}{N} = \frac{1}{100} = 0.01$ mm

Zero error = 4 × Least, count = 0.04 mm

Observed diameter = 4+ (35 × 0.01) = 4.35 mm

Corrected diameter = (4.35 - 0.04) mm = 4.31 mm

As $V = \pi\left(\frac{D}{2}\right)^2 l$

$$\therefore \left(\frac{\Delta V}{V}\right)_{\max} = \frac{2\Delta D}{D} + \frac{\Delta l}{l} = 2\times\frac{0.01}{4.31}\times 100\% + 2.25\% = 2.71\%$$

2. **(a)**

For downward motion,

$h_1 = ut - \frac{1}{2}gt^2$ [$\because a = -g$]

$h_1 = 5u - 125$ … (i)

For downward motion,

$h_2 = ut + \frac{1}{2}gt^2$ [$\because a = +g$]

$h_2 = 5u + 125$ … (ii)

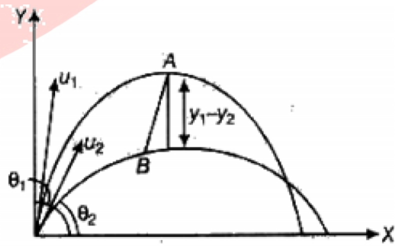
$\therefore h = h_1 + h_2$

$= (5u - 125) + (5u + 125)$ [from Eqs. (i) and (ii)]

$= 10u$

3. **(d)**

The instantaneous positions of the two balls are given by (in y-direction)



Ball A, $y_1 = (u_1 \sin\theta_1)t - \frac{1}{2}gt^2$

Ball B, $y_2 = (u_2 \sin\theta_2)t - \frac{1}{2}gt^2$

The position of the ball A with respect to ball B is given by

$$y = y_1 - y_2 = (u_1 \sin\theta_1 - u_2\sin\theta_2)t = (50\sin 60 - 20\sin 30)\times 3 = 100\,\text{m}$$

4. **(b)**

Velocity of a block at point B is,

$$v = \sqrt{2gh} \qquad \ldots \text{(i)}$$

To complete the vertical circle, the velocity of block should be atleast $\sqrt{5gR}$

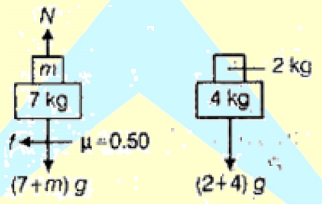
$\therefore \quad v \geq \sqrt{5gR}$

$\Rightarrow \quad \sqrt{2gh} \geq \sqrt{5gR} \qquad$ [from Eq. (i)]

$\Rightarrow \quad h \geq \dfrac{5R}{2} \qquad \Rightarrow \qquad h_{\min} = \dfrac{5R}{2} = \dfrac{5 \times 4}{2} = 10 \text{ cm}$

5. **(a)**

FBD of given system is



$N = (7+m)g$

$f = \mu N = 0.50\,(7+m)g$

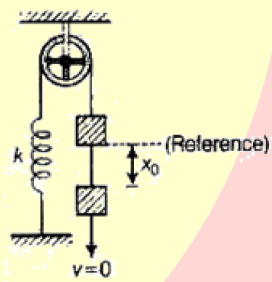
For the blocks to remain at rest

$6g \leq 0.50(7+m)g$

$12 \leq (7+m)$

$m \geq 5 \Rightarrow m_{\min} = 5\,\text{kg}$

6. **(a)**



When block goes down by x_0, Spring extends by x_0

Loss in gravitational potential of the block = gain in energy of spring.

$mgx_0 = \dfrac{1}{2}kx_0^2 \Rightarrow k = \dfrac{2mg}{x_0}$

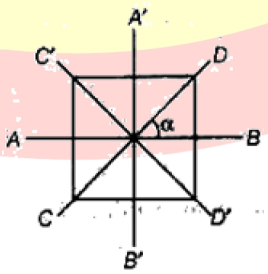
Maximum extension of spring

$= \dfrac{2mg}{k} = \dfrac{10g}{k} = \dfrac{10 \times 10}{2} = 50\,\text{m}$

7. **(a)**

$A'B' \perp AB$ and $C'D' \perp CD$

From symmetry,



$I_{AB} = I_{A'B'} = 2I$ and $I_{CD} = I_{C'D'}$

From theorem of perpendicular axes,

$I_{ZZ} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$

$I_{AB} + I_{A'B'} = 2I + 2I = 4I$

$\Rightarrow I_{ZZ} = 4I$

$\Rightarrow\ 4I = I_{CD} + I_{C'D'}$

$\Rightarrow\ 4I = 2I_{CD}$ $\quad [\because I_{CD} = I_{C'D'}]$

$\Rightarrow\ I_{CD} = 2I$

$\Rightarrow\ I_{AB} = I_{CD} = 2I$

8. **(a)**

Angular momentum

= Mass × Orbital velocity × Radius

$= 2m\left(\sqrt{\frac{GM}{R_0}}\right) \times R_0 = 2m\sqrt{GMR_0}$

9. **(b)**

Pressure at half the depth $= p_0 + \frac{h}{2}\rho g$

Pressure at the bottom $= p_0 + h\rho g$

According to given condition,

$$p_0 + \frac{h}{2}\rho g = \frac{2}{3}(p_0 + h\rho g)$$

$$\Rightarrow\quad 3p_0 + \frac{3h}{2}\rho g = 2p_0 + 2h\rho g \quad \Rightarrow \quad h = \frac{2p_0}{\rho g} = \frac{2\times10^5}{10^3\times10} = 20\,\text{m}$$

10. **(a)**

Ice will absorb heat to raise its temperature to 0°C, then its melting takes place.

Let m_i = Initial mass of ice

m_i' = Mass of ice that melts

m_w = Initial mass of water

By law of mixture, heat gained by ice = heat lost by water

$\Rightarrow\quad m_i \times C \times (13) + m_i' \times L = m_w C_w (13)$

$\Rightarrow\quad (3)(0.5)(13) + m_i' \times 80 = (10)(1)(13)$

$\Rightarrow\quad 80m_i' = 100.5$

$m_i' = 1.4\,\text{kg}$

$\therefore$ Final mass of water = Initial mass of water + Mass of ice that melts = 10 + 1.4 = 11.4 kg

11. **(b)**

As $pV = \mu RT = \frac{m}{M}RT \Rightarrow \frac{m}{Vp} = \frac{M}{RT} \Rightarrow \frac{\text{density}}{p} = \frac{M}{RT}$

$$\left(\frac{\text{density}}{p}\right)_{\text{At } 27°C} = \frac{M}{R(300)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{M}{R} = \frac{300}{2} \qquad \dots \text{(i)}$$

$$\left(\frac{\text{density}}{p}\right)_{\text{At } 153°C} = \frac{M}{R(426)} \qquad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\left(\frac{\text{density}}{p}\right)_{\text{At } 153°C} = \frac{300}{2 \times 426} = \frac{75}{213}$$

12. **(d)**
13. **(a)**
14. **(a)**

Using Charle's law, we have

$$\frac{V}{T} = \text{constant}$$

$$\Rightarrow \frac{A\left(\frac{l}{2}+10\right)}{363} = \frac{A\left(\frac{l}{2}-10\right)}{300} \Rightarrow \frac{\frac{l}{2}+10}{363} = \frac{\frac{l}{2}-10}{300}$$

As the piston moves 10 cm, the length of one side will be $\left(\frac{l}{2}+10\right)$ and other side $\left(\frac{l}{2}-10\right)$.

$$\Rightarrow 300\left(\frac{l}{2}+10\right) = 363\left(\frac{l}{2}-10\right) \Rightarrow 150l + 3000 = 181.5l - 3630$$

$$\Rightarrow 6630 = 31.5l \Rightarrow l = 210.4\,\text{cm}$$

15. **(b)**

Given equation of stationary wave is

$$y = \sin 2\pi x \cos 2\pi t$$

Comparing it with standard equation,

$$y = 2A \sin\frac{2\pi x}{\lambda} \cos\frac{2\pi t}{\lambda}$$

We have, $\frac{2\pi x}{\lambda} = 2\pi x \Rightarrow \lambda = 1\text{ m}$

Minimum distance of string (first mode)

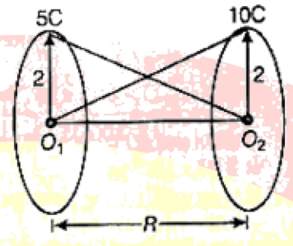
$$L_{\min} = \frac{\lambda}{2} = \frac{1}{2}\text{m}$$

16. **(b)**
17. **(c)**

18. **(b)**

$W = q(V_{02} - V_{01})$ where, $V_{01} = \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{Q_2}{4\pi\varepsilon_0 R\sqrt{2}}$

$$V_{02} = \frac{Q_2}{4\pi\varepsilon_0 R} + \frac{Q_1}{4\pi\varepsilon_0 R\sqrt{2}}$$



$$\Rightarrow \quad V_{02} - V_{01} = \frac{(Q_2 - Q_1)}{4\pi\varepsilon_0 R}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \quad W = \frac{q(Q_2 - Q_1)}{4\pi\varepsilon_0 R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$$

On putting the values,

$Q_1 = 5C,\ Q_2 = 10C,\ R = 2\text{m},\ q = 3C,$ we get

$W = 2.2k$

19. **(b)**

20. **(d)**

$$B = \frac{\mu_0 \theta_i}{4\pi r} \Rightarrow B \propto \theta_i$$

But $\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}$ … (i)

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \cdot \frac{i_1}{i_2}$$

So, $\frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_1}$ [from Eq. (i)]

$$\Rightarrow \quad B_1 = B_2$$

21. **(d)**

If charged particle is moving perpendicular to the direction of **B**, it experiences a maximum force which acts perpendicular to the direction of **B** as well as **v**. Hence, this force will provide the necessary centripetal force and the charged particle will move in a circular path in the magnetic field of radius r and is given by

$$\frac{mv^2}{r} = qvB$$

Kinetic energy of electron = 20 eV

$$\Rightarrow \quad \frac{1}{2}mv^2 = 20 \text{ eV}$$

$\therefore \quad \frac{1}{2}\left(9.1\times10^{-31}\right)v^2 = 20\times1.6\times10^{-19}$

$\Rightarrow \quad v^2 = \frac{2\times20\times1.6\times10^{-19}}{9.1\times10^{-31}}$

$\Rightarrow \quad v^2 = 7.03\times10^{12}$

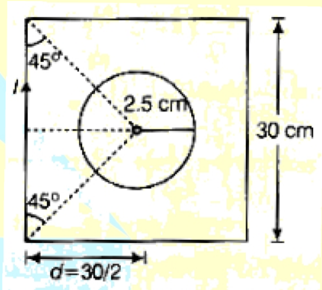
$\Rightarrow \quad v = 2.65\times10^{6}\text{ m/s}$

Now, radius of circular path,

$$r = \frac{mv}{qB} = \frac{9.1\times10^{-31}\times2.65\times10^{6}}{1.6\times10^{-19}\times3\times10^{-5}} = 50\text{ cm}$$

22. **(b)**

Magnetic field due to square loop at centre will be



$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\cos\theta_1 + \cos\theta_2\right)$$

$$B_1 = \frac{\mu_0 I}{4\pi\frac{L}{2}}\left(\cos45° + \cos45°\right) = \frac{\mu_0 I}{\sqrt{2}\pi L}$$

where, $L =$ side of square loop

$I =$ current flowing through square loop

$$B_0 = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

Flux through smaller loop,

$$\phi = B_0\left(\pi r^2\right) = \frac{2\sqrt{2}\mu_0 I\pi r^2}{\pi L} = MI$$

$$\therefore \quad M = \frac{2\sqrt{2}\mu_0 r^2}{L}$$

Here, $r =$ radius of circular loop

On putting values of r and L, we get

$$M = \frac{2\sqrt{2}4\pi\times10^{-7}\times(0.025)^2}{(0.30)} = 7.4\times10^{-9}\,H$$

23. **(c)**

24. **(b)**

25. **(c)**

26. **(d)**

In this case, for photoelectric emission the wavelength of incident radiations must be less than 5200 Å. Wavelength of ultraviolet radiations is less than this value (5200 Å) but wavelength of infrared or blue radiations is higher than this value.

27. **(a)**

$\beta = 50,\ R_i = 4k\Omega = 4000\Omega,\ V_i = 0.02V,\ V_i = \Delta I_B R_i$

$$\Rightarrow \quad \Delta I_B = \frac{V_i}{R_i} = \frac{0.02}{4000} = 5\mu A$$

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

$$\Rightarrow \quad \Delta I_C = \beta \Delta I_B = 50 \times 5 \times 10^{-6} A = 2.5 \times 10^{-4} = 0.25 \times 10^{-3} = 0.25\, mA$$

28. **(c)**

29. **(c)**

Distance of nth minima from central bright fringe,

$$x_n = \frac{(2n-1)\lambda D}{d}$$

For $n = 3$, *i.e.* 3rd minima,

$$x_3 = \frac{(2\times 3-1)\times 500\times 10^{-9}\times 1.5}{2\times 0.5\times 10^{-3}} = 3.75\times 10^{-3}\text{m} = 3.75\,\text{mm}$$

30. **(a)**

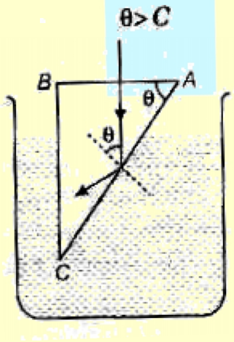
For total internal reflection at AC

$$\Rightarrow \quad \sin\theta \geq \sin C$$

$$\Rightarrow \quad \sin\theta \geq \frac{1}{{}_w\mu^g}$$

$$\Rightarrow \quad \sin\theta \geq \frac{\mu_w}{\mu^g}$$

$$\Rightarrow \quad \sin\theta \geq \frac{8}{9}$$



CHEMISTRY

31. **(c)**

19[th] electron of chromium is $4s^1$ $n = 4, l = 0, n = 0, s = +\frac{1}{2}$

32. **(d)** 33. **(c)** 34. **(d)** 35. **(c)**

36. **(c)**

Superoxides (O_2^-) have one unpaired electron each and are, therefore, paramagnetic. Oxide ions (O^{2-}) and peroxides (O_2^{2-}) donot have any unpaired electrons and hence are diamagnetic.

37. **(d)**

C_2H_5I and C_2H_5OH do not form ideal solution.

38. **(a)**

$P = \frac{n^2 a}{V^2}; a = \frac{PV^2}{n^2} =$ atm dm^6 mol^{-2}

39. **(a)**
40. **(c)**
41. **(a)**

NH_4Cl and NH_4OH is a buffer solution (weak base and salt of strong acid)

42. **(a)**
43. **(c)**

Velocities of both K^+ and NO_3^- are nearly the same in KNO_3 so it is used to make salt-bridge.

44. **(c)**

$CICH_2COONa + HCl \rightarrow CICH_2COOH + NaCl$

$\lambda_{CICH_2COONa} + \lambda_{HCL} = \lambda_{CICH_2COOH} + \lambda_{NaCl}$

$224 + 203 = \lambda_{CICH_2COOH} + 38.2$

$\lambda_{CICH_2COOH} = 427 - 38.2 \quad = 388.8\ ohm^{-1} cm^2 gm\ eq^{-1}$

45. **(b)**

Gold number $\propto \frac{1}{protective\ power}$

46. **(d)**

As we go down the group inertness of ns^2 pair increase hence tendency to exhibit +2 oxidation state increase and that of +4 oxidation state decreases.

47. **(b)**
48. **(c)**

$KF + HF \longrightarrow KHF_2 \rightleftharpoons K^+ + HF_2^-$

49. **(b)**

Lithium form nitride on heating with nitrogen. Lithium nitride gives ammonia when heated with H_2O. Ammonia gas form tetrammine copper complex with $CuSO_4$ solution.

$6Li + N_2 \longrightarrow 2Li_3N$

$Li_3N + 3H_2O \longrightarrow 3LiOH + NH_3$

$CuSO_4 + 4NH_3 \longrightarrow \left[Cu\left(NH_3\right)_4\right]SO_4$

50. **(b)**

O.N. of Fe is + 1 : x + 5(0) + 0 = + 2 or x = + 2.

51. **(d)**

$[Pt(en)_2Cl_2]$ will show two geometcial forms and cis form will show optical isomerism.

52. **(b)** 53. **(b)** 54. **(c)**

55. **(d)**

$CH_3CH_2\text{–}\overset{\overset{\displaystyle O}{\|}}{C}\text{–}H$, $CH_3\text{–}\overset{\overset{\displaystyle O}{\|}}{C}\text{–}CH_3$, $CH_2{=}CH\text{–}O\text{–}CH_3$, $CH_2{=}CH\text{–}CH_2OH$

56. **(d)** 57. **(c)** 58. **(d)** 59. **(d)**

60. **(a)**

MATHS

61. **(b)**

$g\ o\ f(x) = g(f(x)) = \frac{1}{2}\sin[(x)]\pi = 0,$ for all $x \in R.$ Hence the range of $g\ o\ f$ is $\{0\}$

62. **(b)**

$f(0) = \lim_{x\to 0} f(x) \ = \lim_{x\to 0}\left[a\log(1+ax)^{1/x} + b\log(1-bx)^{-1/bx}\right] = a+b$

63. **(a)**

f is continuous except possibly $x=1$ and $x=-1$ $\lim_{x\to 1-} f(x) = a+b$ and $\lim_{x\to 1+} f(x)1$ so $a+b=1$ $f(1+)=-1,$ $f'(1-)=2a.$ Hence $a=-1/2, b=3/2.$

64. **(c)**

Put $x=\sin(\cos\theta)$ so $y=\tan\theta/2.$ Thus $\frac{dy}{dx} = \left(\frac{1}{2}\sec^2\theta/2\right)\left(-\frac{1}{\sin\theta(\cos\theta)}\right) \Rightarrow y'(0) = -1$

65. **(b)**

66. **(b)**

$\int_0^a f(x)g(x)h(x)dx = \int_0^a f(a-x)g(a-x)h(a-x)dx \ = -\frac{1}{4}\int_0^a f(x)g(x)[3h(x)-5]dx$

67. **(c)**

The given equation can be written as

$\frac{xdy-ydx}{x^2+y^2} = -dx \Rightarrow \frac{xdy-ydx}{x^2}\times\frac{1}{1+y^2/x^2} = -dx \ \Rightarrow \frac{1}{1+y^2/x^2}\frac{d}{dx}(y/x) = -dx.$ Integrating we have

$\tan^{-1}(y/x) = -x+c \ \Rightarrow \ y = x\tan(c-x)$

68. **(a)**

Put $\sqrt{x}=t$ or $x=r^2,$ so that

$I = 2\int_0^\infty \frac{t^2}{(1+t^2)(2+t^2)(3+t^2)}dt \ = \int_0^\infty\left(-\frac{1}{1+t^2}+\frac{4}{2+t^2}-\frac{3}{3+t^2}\right)dt$

$= \left(-\tan^{-1}t + \frac{4}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{3}{\sqrt{3}}\tan^{-1}\left(\left(\frac{t}{\sqrt{3}}\right)\right)\right]_0^\infty \ = -\frac{\pi}{2} + 2\sqrt{2}\left(\frac{\pi}{2}\right) - \sqrt{3}\left(\frac{\pi}{2}\right) \ = \frac{\pi}{2}\left(2\sqrt{2}-\sqrt{3}-1\right)$

69. **(b)**

$p_1 = \frac{m^2+2m+1}{\sqrt{2}}, p_2 = \frac{mm'+m+m'+1}{\sqrt{2}}$ $p_3 = \frac{m'2+2m'+1}{\sqrt{2}}$ $\Rightarrow p_1p_3 = p_2^2 \Rightarrow p_1, p_2, p_3$ are in G.P.

70. **(c)** 71. **(d)** 72. **(d)**

73. **(d)**

Equations of the normal at P is $ax+by \text{ cosec } \theta = (a^2+b^2)\sec\theta$ ... (1)

And the equation of the normal at Q $(a \sec\phi, b\sec\phi)$ is $ax+by \text{ cosec}\phi = (a^2+b^2)\sec\phi$ ... (2)

Subtracting (2) from (1) we get

$$y = \frac{a^2+b^2}{b} \cdot \frac{\sec\theta - \sec\theta}{\text{cosec}\,\theta - \text{cosec}\,\theta}$$

So that $k = y = \frac{a^2+b^2}{b} \frac{\sec\theta - \sec(\pi/2-\theta)}{\text{cosec }\theta - \text{cosec}(\pi/2-\theta)}$ $[\because \theta+\phi = \pi/2]$

$$= \frac{a^2+b^2}{b} \frac{\sec\theta - \text{cosec}\,\theta}{\text{cosec}\,\theta - \sec\theta} = -\left[\frac{a^2+b^2}{b}\right]$$

74. **(c)**

Equation of the tangent at $(a \cos\theta, b \sin\theta)$ to the ellipse $x^2/a^2 + y^2/b^2 = 1$ is

$$\Rightarrow \quad \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \quad \text{... (1)}$$

The joint equation of the lines joining the points of intersection of (1) and the auxillary circle $x^2+y^2=a^2$ to the origin, which is the centre of the circle, is

$$x^2+y^2 = a^2\left[\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right]^2$$

Since these lines are at right angles

Co-efficient of x^2 + Co-efficient of $y^2 = 0$

$$\Rightarrow 1-a^2\left(\frac{\cos^2\theta}{a^2}\right)+1-a^2\left(\frac{\sin^2\theta}{b^2}\right)=0 \Rightarrow \sin^2\theta\left(1-\frac{a^2}{b^2}\right)+1=0$$

$$\Rightarrow \sin^2\theta(b^2-a^2)+b^2=0 \Rightarrow \sin^2\theta\left[a^2(1-e^2)-a^2\right]+a^2(1-e^2)=0$$

$$\Rightarrow (1+\sin^2\theta)a^2e^2 = a^2 \quad \Rightarrow \quad e = (1+\sin^2\theta)^{-1/2}$$

75. **(a)**

Let the coordinate of A, B, C be $(t_i^2, 2t_i)$ $i = 1,2,3$ respectively. The tangents at A and B are $t_1 y = x + t_1^2$ and $t_2 y = x + t_2^2$

Which intersect at $x = t_1 t_2, y = t_1 + t_2$

So the vertices are $P(t_1t_2,\ t_1+t_2), Q(t_1t_3, t_1+t_3)$ and $R(t_1t_3, t_1+t_3)$

$\Delta = \left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$

$$\Delta' = \left\| \frac{1}{2} \begin{vmatrix} t_1 t_2 & t_1 + t_2 & 1 \\ t_2 t_3 & t_2 + t_3 & 1 \\ t_3 t_1 & t_3 + t_1 & 1 \end{vmatrix} \right\| = \left\| \frac{1}{2} \begin{vmatrix} (t_1 - t_3)t_2 & t_1 - t_3 & 0 \\ (t_2 - t_1) & t_2 - t_1 & 0 \\ t_3 t_1 & t_3 + t_1 & 1 \end{vmatrix} \right\| = \left| \frac{1}{2}(t_1 - t_3)(t_2 - t_1)(t_2 - t_3) \right| \Rightarrow \Delta = 2\Delta'$$

76. **(b)**

77. **(b)**

Produce the join of z_1 and z_2 to meet $a\bar{z} + a\bar{z} + b = 0$ in z. Suppose z divides the join of z_1 and z_2 externally in the ratio $k : 1$ where $k > 0$, so that

$$z = \frac{z_1 - kz_2}{1 - k}$$

As z lies on $\bar{a}z + a\bar{z} + b = 0$

$$\bar{a}(z_1 - kz_2) + a(\bar{z}_1 - k\bar{z}_2) + b(1 - k) = 0 \Rightarrow \frac{\bar{a}z_1 + a\bar{z}_1 + b}{\bar{a}z_2 + a\bar{z}_2 + b} = k > 0$$

78. **(b)**

$$(z+1)(z^2 - z + 1) + 2z(z+1) = 0 \Rightarrow (z+1)(z^2 + z + 1) = 0 \Rightarrow z = -1, \omega, \omega^2$$

Out of these ω and ω^2 satisfy

$$z^{2006} + z^{100} + 1 = 0$$

79. **(a)**

$$D = (b^2 + a^2 - c^2)^2 - 4a^2b^2$$

$$= (a^2 + b^2 - 2ab - c^2)(a^2 + b^2 - c^2 + 2ab) = \left[(a-b)^2 - c^2\right]\left[(a+b)^2 - c\right] > 0$$

80. **(c)**

$$\frac{1}{19}S = \frac{4}{19^2} + \frac{44}{19^3} + \ldots$$

Subtract form S to obtain

$$\frac{18}{19}S = \frac{4}{19^2} + \frac{44}{19^3} + \ldots$$

Subtract form S to obtain

$$\frac{18}{19}S = \frac{4}{19} + \frac{40}{19^2} + \frac{400}{19^3} + \ldots.$$

81. **(b)**

$$\Delta = \begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = -3(3 + \lambda)$$

For $\lambda = -3$, sum of the last two equation gives $2x - y - 2z = 0$. Compare itwith the first equation.

82. **(d)**

The number of wave of choosing 6 persons out of 12 for a group is ${}^{12}C_6$.

The number of ways in which this group can contains 3 boys and 3 girls is $\left({}^{6}C_3\right)\left({}^{6}C_3\right)$.

Therefore required probability $=\dfrac{\left({}^{6}C_3\right)\left({}^{6}C_3\right)}{{}^{12}C_6}=\dfrac{100}{231}$.

83. **(a)**

Let A denote the event that the runner succeeds exactly 3 times out of five and B denote the event that the runner succeeds on the first trial.

$$P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}$$

But $P(B\cap A)=P$ (succeeding in the first trial and exactly once in two other trials)

$=p\left({}^{4}C_2\,p^2(1-p)^2\right)=6p^3(1-p)^2$ and $P(A)={}^{5}C_3p^3(1-p)^2=10p^3(1-p)^2$

Thus, $P(B\,|\,A)=\dfrac{6p^3(1-p)^2}{10p^3(1-p)^2}=\dfrac{3}{5}$.

84. **(b)**

85. **(c)**

$$\frac{2R(\sin A\cos A+\sin B\cos B+\sin C\cos C)}{2R(\sin A+\sin B+\sin C)}$$

$$=\frac{(\sin 2A+\sin 2B+\sin 2C)}{2(\sin A+\sin B+\sin C)}=\frac{4\sin A\sin B\sin C}{8\cos(A/2)\cos(B/2)\cos(C/2)}=4\sin(A/2)\sin(B/2)\sin(C/2)=\frac{r}{R}$$

86. **(a)**

$$\frac{1+\tan\theta_1\tan\theta_2}{1-\tan\theta_1\tan\theta_2}+\frac{1-\tan\theta_3\tan\theta_4}{1+\tan\theta_3\tan\theta_4}=0\ \Rightarrow\ 2+2\tan\theta_1,\tan\theta_2\tan\theta_3\tan\theta_4=0$$

87. **(b)**

88. **(c)**

Volume of the parallelopiped formed by $i+aj+k,\ j+ak+k$ is

$$V=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a$$

$$\frac{dV}{da}=3a^2-1$$

For minimum value of V, we have $\dfrac{dV}{da}=0$

$\Rightarrow a=\pm 1\sqrt{3}$ Also $\dfrac{d^2V}{da^2}=6a>0$ for $a=1/\sqrt{3}$. Thus V is minimum when $a=1/\sqrt{3}$.

89. **(b)**

The line is parallel to the vector $i+j+k$ and the normal to the plane is $3i-4j+5k$, so that angle between these vectors is $\pi/2-\theta$

$$\cos(\pi/2-\theta)=\frac{(i+j+k)(3i-4i+5k)}{\sqrt{1+1+1}\sqrt{3^2+4^2+5^2}} \Rightarrow \sin\theta=\frac{3-4+5}{\sqrt{3}\sqrt{50}}=\frac{2\sqrt{6}}{15}$$

90. **(d)**

The given lines intersect, if the shortest distance between the lines is zero.

We know that the shortest distance between the line $r=a_1+\lambda b_1$ and $r=a_2+\mu b_2$ is

$$\frac{|(a_1-a_2)b_1\times b_2|}{|b_1\times b_2|}$$

So the shortest distance between the given lines is zero if

$$(i-j-(2i-j)).(2i+k)\times(i+j-k)=0$$

$$\text{L.H.S.}=\begin{vmatrix}-1 & 0 & 0\\ 2 & 0 & 1\\ 1 & 1 & -1\end{vmatrix}=1\neq 0$$

Hence the given lines do not intersect.